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Chapter 1

Historical Studies of Hellin’s Law

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Abstract

Theorems, proofs, laws and rules are commonly named according to the presumed discoverer, but often earlier investigators have contributed substantially to the findings. One example of this is Hellin’s law, which was named after Hellin, although he was not the first to derive it. In research on twinning and higher multiple maternities, the law has played a central role because it is approximately correct, despite showing discrepancies that are difficult to explain or eliminate. However, most studies are based on empirical rates of multiple maternities. Such studies can only serve to identify errors too large to be characterized as random. It has been mathematically proven that Hellin’s law does not hold as a general rule. Consequently, improvements to this law have been proposed.

Keywords: twinning rates, triplet rates, quadruplet rates, Stigler’s law, maternal age, temporal trends

1. Introduction

During the history of research on multiple maternities, Hellin’s law has been applied as a rule of thumb. Consequently, the law contributes to the description of the twinning models. In this paper, we consider how Hellin’s law can be tested and used. It is of particular interest to determine why the rates of higher multiple maternities are sometimes too high or too low when Hellin’s law is used as a benchmark. The analysis of Fellman and Eriksson [1] of triplet and quadruplet rates indicated that triplet rates are closer to Hellin’s law than quadruplet rates. According to the analyses by Fellman and Eriksson [2] of the twinning rate and the transformed triplet rate and quadruplet rate for Sweden (1751–2000), both triplet and quadruplet rates showed excesses after the 1960s. This is mainly caused by the influence of the artificial reproduction technologies, particularly the use of fertility-enhancing drugs. Fellman and Eriksson [2] introduced measures of concordance between triplet rates with Hellin’s law.

Regression analyses of twinning and triplet rates yield rather good fits with respect to Hellin's law, but deficiencies in the triplet rates are commonly present. According to Hellin's law, historical data show deficiencies in triplet rates, but recent data reveal excesses, especially among older mothers. The excesses obtained are in good agreement with other studies of recent data. Here, we pay special attention to the use of Hellin's law in investigations of multiple maternities.

2. Prerequisites for twinning research

In the nineteenth century, a series of statistical congresses, most notable in Brussels in 1853 and in St. Petersburg in 1872, was important for the start of demographic research and especially twinning research [3]. Levi [4] gave a detailed presentation of the suggestions accepted at the congress in Brussels.

> *[T]here ought to be an annual registry of population, exhibiting the births by sex, by age of both parents, legitimate and illegitimate, number of twins, stillborn, marriages and divorces, by months. The deaths, by sex, by age, and by months, distinguishing among dead children, till three years of age, the legitimate from the illegitimate. The deaths by month, with the causes of death, and the profession of the deceased; marriages, with the age of the parties, their condition, profession, and number of children, distinguishing the legitimate and those acknowledged as such. Considering the extreme importance of a uniform nomenclature of diseases equally applicable to all countries, the attention of learned men is to be called to the question for further consideration at some future congress.*

According to Brown [5], the principal discussion at the St. Petersburg congress centred around facts relating to the movement of the population and the mode in which they should be registered. Among the facts to be registered were in multiple maternities the sex and number of the children, stillborn or born alive, whether legitimate or not and the age and parity of the mother on the birth date.

Westergaard [6] has devoted a whole chapter in his history of statistics to the presentations of the statistical congresses in the middle of the nineteenth century and their importance [1]. The first congress was held in Brussels in 1853. Brussels was chosen as the first meeting place because the Belgian Central Commission undertook the great preparatory work. The congress was held under the presidency of Adolphe Quetelet. The aim of the congress being chiefly a practical one, there was no room for lectures on special scientific problems. During congresses after Brussels, discordance increased among the European countries, and thus, the International Statistical Congress came to an end.

Recently, Droesbeke [7] gave a short but detailed presentation of the planning and findings of the Brussels congress and subsequent international statistical congresses. His text agrees the ideas of Westergaard [6]. A commission was created in March 1841 as a first step towards the realization of the congress. The president of the Central Commission of Statistics, Adolphe Quetelet, played a central role in the planning process. Quetelet was held in high esteem in London since 1833, when he had taken part in the congress of the British Association for the

Advancement of Science, at the end of which he had contributed to the creation of a section devoted to statistics. The idea to organize a congress, the main objective of which was to lay down a common foundation for statistical data to enable international comparisons, was presented by Quetelet 1851. He offered to

> *bring together at a congress in Brussels the persons, who in various countries, are dealing specifically with statistics, in order to give their works a common impulse, and to adopt, for the computations, uniform grounds which will allow for comparison of observations and results. (Bulletin of the Commission, 1853, p. 106).*

Furthermore, Droesbeke describes the congress programme and stressed the content of the congress according to Quetelet's words:

> *It is to be hoped that the works which belongs to this science will now be taken on, in every State, following the bases which have been laid down during the Congress of Brussels. It is not any more a theoretical wish, to see the states adopt uniform bases for scientific works in order to make the results comparable; the opportunity of implementation of the idea has been proclaimed; the framework has been chosen and the reading of the report […] will demonstrate what is allowed to expect from the wisdom, maturity, perfect intelligence and good harmony which presided over the deliberations of the Congress.*

Finally, Droesbeke listed the Statistical Congresses during the nineteenth century that followed the first one in Brussels.

Recently, Randeraad [8] directed attention to and even criticism of the international statistical congresses in the second half of the nineteenth century. He stated that it would be overly simplistic to assume that they were an outright success. In fact, no more congresses were held after 1876. Furthermore, he stressed that:

> *More importantly, by then it was clear that the aspirations of the early congresses had been too high. International uniformity in statistics was evidently not a goal that could be reached overnight. Much of this failure to bring about rapid change can be explained by the difficulties in realizing effective knowledge transfers, in other words effective communication, in an age that was not fully prepared for truly international activities. It has been shown that the second half of the nineteenth century was a period of numerous experiments in internationalism, but at the same time rampant nationalism nipped many initiatives in the bud.*

In most countries, the registers were deemed lacking in essential facts; those of Belgium and Sweden were perhaps the most detailed for scientific inquiries [3]. Arosenius [9] presented a detailed study of the emergence of the official statistics of Sweden. His presentation shows just how difficult the development of the process is until an official statistics of modern proficiency is born.

Already in the eighteenth century, Wargentin published demographic data for Sweden. However, he did not pay any attention to twinning and higher multiple maternities [10]. Berg [11] published a comprehensive study of multiple maternities. He analyzed the rates of multiple maternities in Sweden from 1776 to 1878. He also presented corresponding data for several

European countries and analyzed the sex combinations of twin, triplet and quadruplet sets in Sweden from 1869 to 1878. His study was published in Swedish, and thus, few scientists were aware of this paper. Since Swedish is the native language of our group, Berg's results have been of great value in our studies [1, 12–14].

During the second half of the nineteenth century, Statistics Sweden published in the journal Statistisk Tidskrift an extensive time series of demographic data. The data were given separately for different counties of Sweden and contained the size of the population, the number of births (live and stillborn) and twin, triplet and quadruplet sets. A list of these data was given in **Table 1** in [1], indicating that Sweden has overall the oldest continuous population statistics worldwide. Our group has used these data in different studies [1, 15].

County (Län)	Period	Reference
Stockholm city	1749–1858	ST, 1860–62:43–47
Stockholm county	1749–1773, 1795–1858	ST, 1860–62:134–141
Uppsala	1749–1773, 1795–1859	ST, 1860–62:280–288
Södermanland	1749–1773, 1795–1859	ST, 1860–62:317–324
Östergötland	1749–1773, 1795–1860	ST, 1863–65:164–171
Jönköping	1749–1773, 1795–1862	ST, 1863–65:266–273
Kronoberg	1749–1773, 1795–1862	ST, 1863–65:274–281
Kalmar	1749–1773, 1795–1868	ST, 1870:211–220
Gotland	1759–1869	ST, 1870:27:221–231
Blekinge	1749–1773, 1795–1869	ST, 1870:232–240
Kristianstad	1749–1773, 1795–1871	ST, 1873:133–142
Malmöhus	1749–1773, 1795–1871	ST, 1873:143–152
Halland	1749–1773, 1795–1871	ST, 1873:153–162
Göteborg and Bohus	1749–1773, 1795–1859	ST, 1860–62:388–400
Älvsborg	1749–1773, 1795–1874	ST, 1875:127–136
Skaraborg	1749–1773, 1795–1876	ST, 1877:156–168
Värmland	1795–1865	ST, 1877:170–176
Örebro (Närke)	1749–1773	ST, 1877:166–169
Västmanland	1749–1773, 1795–1887	ST, 1888:159–170
Kopparberg	1749–1773, 1795–1887	ST, 1888:171–182
Gävleborg	1749–1773, 1795–1887	ST, 1888:161–172
Västernorrland	1792–1888	ST, 1888:173–184
Jämtland	1792–1888	ST, 1888:185–196
Västerbotten	1802–1860	ST, 1863–65:50–57
Norrbotten	1802–1860	ST, 1863–65:44–49

Table 1. The Division of Sweden into 25 counties for regional data concerning population size, births and multiple maternities, 1749–1888 (ST = Statistisk Tidskrift) [1].

3. Genesis of Hellin's law

The Veit data set from Prussia (1826–1849), presented by Fellman and Eriksson [1] in **Table 2**, consists of 13,360,557 maternities, including 13,208,868 single, 149,964 twin, 1689 triplet and 36 quadruplet maternities [16]. Veit analyzed the temporal trend in the twinning rate (TWR) and noted very small variations, but during the first half of the period, the annual TWRs were almost constantly higher than during the last half of the period (except for the year 1849). The trend may be seen elsewhere ([1], **Table 2** and **Figure 1**). For the total data set, Veit noted the following rates: for twin pairs 1:89, for triplet sets 1:7910 and for quadruplet sets 1:371126. He did not give the relations between TWR, triplet rate (TRR) and quadruplet rate (QUR), that is,

	Maternities				
Year	**All**	**Single**	**Twin**	**Triplet**	**Quadruplet**
1826	519,633	513,727	5824	80	2
1827	485,165	479,724	5374	65	2
1828	493,749	488,060	5620	69	0
1829	489,604	483,796	5738	69	1
1830	491,659	486,141	5455	62	1
1831	484,889	479,281	5543	65	0
1832	476,035	470,175	5783	76	1
1833	530,954	524,525	6340	87	2
1834	549,750	542,947	6717	83	3
1835	527,148	521,156	5918	73	1
1836	544,177	537,805	6301	69	2
1837	551,450	545,084	6289	77	0
1838	560,086	553,837	6186	61	2
1839	568,487	562,065	6360	59	3
1840	580,747	574,293	6381	72	1
1841	585,085	578,738	6277	67	3
1842	616,845	610,058	6716	71	0
1843	597,912	591,420	6426	64	2
1844	616,287	609,452	6771	59	5
1845	640,214	633,123	7029	60	2
1846	619,727	613,101	6556	69	1
1847	577,007	570,766	6183	58	0
1848	570,737	564,633	6030	73	1
1849	683,210	674,961	8147	101	1
Total	13,360,557	13,208,868	149,964	1689	36

Table 2. Data from Prussia, 1826–1849, according to Veit (1855) [16].

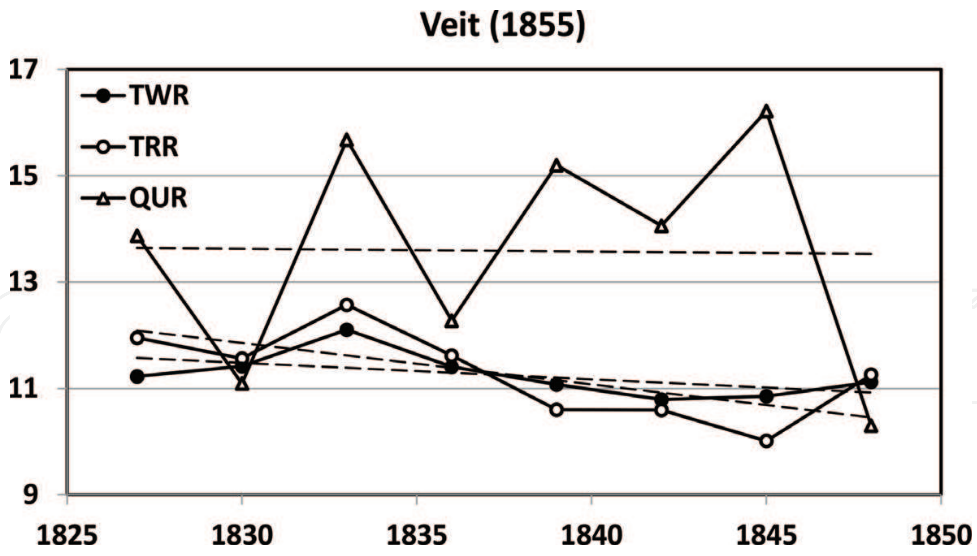

Figure 1. Temporal trends in TWR and transformed TRR and QUR per 10^3 for the Prussian data presented in [16]. Note the excess among QUR.

Hellin's law. He also presented the sex compositions within the twin, triplet and quadruplet sets and noted a lower sex ratio (males to females) among multiple births than among singleton births [1].

The Wappäus data set was collected from different European countries and comprised 19,698,322 maternities, including 226,807 twin and 2623 triplet maternities [1]. Wappäus [17] presented the rates of multiple maternities, but did not discuss the relation between the number of twin, triplet and quadruplet maternities [1].

Bertillon [18] foresaw Hellin's law. He considered multiple maternity data from different countries in central Europe. In his study, he presented the number of triplet maternities per year and per one million total maternities. He also presented the number of total maternities per one triplet maternity and the number of twin maternities per one triplet maternity, i.e., he considered the relation between twin and triplet rates. However, he did not relate the number of total maternities to one twin maternity. Fellman and Eriksson [1] presented a translated version of his table ([18], page 285) and included in columns calculations of the number of total maternities in relation to one twin maternity and the annual mean number of maternities. They believed that had Bertillon included the first of their columns in his table, he would have discovered Hellin's law [1].

Shortly after the congresses in Brussels and St. Petersburg, Neefe [19] published his classical work. He emphasized how important the abovementioned statistical congresses were for the standardization of the demographic registers in different countries, and he used the new possibilities that the improved birth registers offered. Although other contemporaneous studies were published, Fellman and Eriksson [1] stressed that the history of twinning research starts with this publication. Neefe analyzed a long series of problems connected to twinning; these problems have been shown to be central in later studies. He considered inter alia:

1. The rates of twin and higher multiple maternities.
2. The crude birth rates among single and multiple maternities.
3. The regional and seasonal variations in TWRs.
4. The rates of live and stillbirths among twins.
5. The sex composition of sets of multiple maternities.
6. The sex ratio among single and multiple maternities.
7. The effect on the number of multiple maternities of the age of the parents, the marital status and confession of faith of mothers, the residence in urban and rural regions, and the seasonality of the birth.

In addition, he considered weight and prematurity among multiples and mortality among multiples and mothers. This list indicates clearly that Neefe introduced a thorough research programme for twinning studies. It is noteworthy that Neefe did not comment on the relation between the rates of multiple maternities, and consequently, he did not explicitly foretell Hellin's law [1].

Strassmann [20] noted the findings in [16, 17] and concluded, using Veit's total data set, that there is one twin maternity per 89^1 and one triplet maternity per 89^2 total maternities. Strassmann related the number of multiple maternities to the number of all maternities, in contrast to Hellin [21], who related the number of multiple maternities to the number of single maternities. However, both used the same relation, 1:89 [1]. While in the literature authors generally refer to Hellin, they formulate the law according to Strassmann's version. While in the literature authors generally refer to Hellin, the law was already formulated by Strassmann in 1889. Hellin's law has played a central role in the history of research on multiple maternities [3].

Drejer [22] was apparently unaware of Hellin but referred to Strassmann, stating that he had noted the relation between the rates of twin and triplet maternities. Drejer was dubious about the regularity between the rates. He stressed that under such circumstances the rule had to hold also for higher multiple maternities, but he could not find any clear indication of this being the case [1].

Particularly important scientific observations are often associated with a person, but historians of science have, however, noted that often the person associated with a particular finding was not its original discoverer. Scientific observations and results are frequently associated with people who have high visibility and social status, and the results are named long after the discovery. Based on his studies on the history of statistics, Stigler [23] proposed his own *Stigler's law of eponymy*. In brief, the law says: "No scientific discovery is named after its original discoverer". Stigler himself attributes the discovery of Stigler's law to Merton [24], which makes the law self-referencing. Consequently, in this study, one must bear in mind Stigler's law [1].

4. Investigations of Hellin's law

Hellin's law has played a central role in the history of research on multiple maternities. The interest in Hellin's law is mainly the result of its being mathematically simple and approximately correct, but it shows discrepancies that are difficult to explain or eliminate. Statistical studies on empirical rates of multiple maternities can never confirm the law but serve only to identify errors too large to be characterized as random. It is of particular interest to ask why the rates of higher orders of multiple maternities are sometimes too high and sometimes too low when Hellin's law is used as a benchmark [32].

Usually, the arguments for Hellin's law are based on stochastic models for multiple fertilizations and fissions of fertilized eggs. The influence of both multiple fertilizations and fissions of fertilized eggs has inspired scientists to associate the rates of higher multiple maternities with both monozygotic (MZ) and dizygotic (DZ) TWRs (e.g., [25–30]). The contributions by Zeleny [25] have resulted in the law also being known as the Hellin-Zeleny law.

Peller [31] was the first, at least indirectly, to connect Hellin's law to interindividual variation in mothers' chances for multiple maternities. Later, Eriksson [12] considered recurrent twin maternities in families on the Åland Islands (Finland) and presented a modified model (in the paper, the law was called Fellman's law). When Eriksson applied this law to his Åland data, he obtained better congruence with Hellin's law than if Peller's version had been applied. Fellman and Eriksson [1] reviewed papers where the genesis of Hellin's law was traced and where the strengths and weaknesses of the law were analyzed and improvements suggested [32].

Hellin's law presupposes strong correlations between TWR and TRR, but even strong correlations do not prove Hellin's law, establishing only a linear relationship. Fellman and Eriksson [30] considered the correlation between the TWR and the square root of the TRR in Sweden. After elimination of influential temporal factors, they found that the correlation was positive, but not very strong. This finding indicates that, in general, Hellin's law cannot be exact. One application of Hellin's law is to compare TWR and the square root of TRR, the cubic root of QUR and so on [14, 32, 33].

In the following, we consider formulae applicable in the statistical analysis of Hellin's law. Let the theoretical TRR be r. One has different possibilities to study the random errors of the TRR and particularly of the square root of the TRR. The first one is to estimate the standard deviations (SDs) of the TRR and construct confidence intervals (CIs) for r [32].

Let the observed TRR be $\widehat{r}$, then SD $\widehat{r} = \sqrt{\frac{r(1-r)}{n}}$, and the observed standard CI of r is

$$\left(\widehat{r} - \mathrm{k}\sqrt{\frac{\widehat{r}(1-\widehat{r})}{n}},\ \widehat{r} + \mathrm{k}\sqrt{\frac{\widehat{r}(1-\widehat{r})}{n}}\right) \tag{1}$$

where the factor k defines the confidence level. The Hellin-transformed TRR is $\sqrt{\widehat{r}}$. The variance of $\sqrt{\widehat{r}}$ is

$$\mathrm{Var}\left(\sqrt{\widehat{r}}\right) = \left(\frac{d}{dr}\left(\sqrt{r}\right)\right)^2 \mathrm{Var}\left(\widehat{r}\right) = \left(\frac{1}{2\sqrt{r}}\right)^2 \frac{r(1-r)}{n} = \left(\frac{1}{4r}\right)\frac{r(1-r)}{n} = \left(\frac{1-r}{4n}\right) \tag{2}$$

Fellman and Eriksson [34] proposed an alternative transformation $\arcsin\left(\sqrt{r}\right)$. For small values of r, the difference between the two transformations is minute. The variance of $\arcsin\left(\sqrt{\hat{r}}\right)$ is

$$\mathrm{Var}\left(\arcsin\left(\sqrt{\hat{r}}\right)\right) = \left(\frac{\mathrm{d}}{\mathrm{dr}}\arcsin\left(\sqrt{r}\right)\right)^2 \mathrm{Var}\left(\hat{r}\right) =$$
$$\left(\frac{1}{2\sqrt{r}\sqrt{1-r}}\right)^2 \frac{r(1-r)}{n} = \left(\frac{1}{4r(1-r)}\right)\frac{r(1-r)}{n} = \left(\frac{1}{4n}\right) \tag{3}$$

The variance of $\arcsin\left(\sqrt{\hat{r}}\right)$ is slightly larger than the variance of $\sqrt{\hat{r}}$, but it is simpler and does not depend on r. Consider the difference

$$\arcsin\left(\sqrt{r}\right) - \sqrt{r} = \sqrt{r} + \frac{1}{2}\frac{r^{\frac{3}{2}}}{3} + \frac{1\times 2}{3\times 4}\frac{r^{\frac{5}{2}}}{5} + \ldots - \sqrt{r} = \frac{1}{2}\frac{r^{\frac{3}{2}}}{3} + \frac{1\times 2}{3\times 4}\frac{r^{\frac{5}{2}}}{5} + \ldots \approx \frac{1}{6}r\sqrt{r} \tag{4}$$

and $\frac{\arcsin\left(\sqrt{r}\right)-\sqrt{r}}{\sqrt{r}} \approx \frac{r}{6}$. This relative difference between the transformed variables is of the dimension 10^{-4}. The square root is a monotone-increasing function, and consequently, one can construct the CI for $\sqrt{r}$ by a square root transformation of the limits of the CI for r. Hence, for $\sqrt{r}$ the corresponding transformed CI is

$$\left(\sqrt{\hat{r}} - \mathrm{k}\sqrt{\frac{1-\hat{r}}{4n}},\ \ \sqrt{\hat{r}} + \mathrm{k}\sqrt{\frac{1-\hat{r}}{4n}}\right) \tag{5}$$

Fellman and Eriksson [34] gave a mathematical proof that Hellin's law cannot hold in general. If one aggregates heterogeneous data, the fluctuations are smoothed out, but according to Hellin's law, the relation between the TWR and the TRR is not linear, and consequently, the aggregated and disaggregated data cannot simultaneously satisfy Hellin's law [1].

Jenkins [26, 35], Jenkins and Gwin [27], Bulmer [29] and later Fellman and Eriksson [30] have tried to modify the law in order to improve it. Using linear curves is the best method for identifying discrepancies from a presumptive model because graphs containing linear curves are easy to interpret. There are two possibilities for checking Hellin's law with linear curves. One is to use graphs with TWR^2 as abscissa and TRR as ordinate, that is, to use the model $\mathrm{TRR} = \alpha + \beta\mathrm{TWR}^2$. An alternative is graphs with TWR as abscissa and $\sqrt{\mathrm{TRR}}$ as ordinate. Now, the model is $\sqrt{\mathrm{TRR}} = \alpha + \beta\mathrm{TWR}$ [1].

Jenkins and Gwin [27] considered US data for the periods 1923–1924 and 1927–1936. They used TWR^2 as abscissa and TRR as ordinate. From their figure, they obtained the linear relation $\mathrm{TRR} = 0.000013 + 0.656\mathrm{TWR}^2$. The intercept indicated that the line did not pass through the origin and the parameter estimate was markedly below the value one, indicating a deficit in triplet sets. When Fellman and Eriksson [32] applied a regression model to the same data set, they obtained the slightly different result: $\text{TRR} = 0.000039 + 0.584\text{TWR}^2$. The coefficient of determination is $R^2 = 0.842$, indicating a rather good fit. They obtained a deficit in the TRR when they tested the parameter estimate against one with a one-sided t test. The $\text{SE}\left(\widehat{\beta}\right) = 0.113$ yielded $t = -3.7$, and the estimate was significantly below one [1]. As an alternative model, Fellman and Eriksson used TWR as abscissa and $\sqrt{\text{TRR}}$ as ordinate. The estimated model was $\sqrt{\text{TRR}} = 0.0029 + 0.679\text{TWR}$ and $R^2 = 0.844$, $\text{SE}\left(\widehat{\beta}\right) = 0.130$ and $t = -2.5$, and the obtained estimate is significantly below one. Both alternatives indicate deficits in the TRR. The parameter estimates are slightly higher for the first model, but the goodness of fit for both models is comparable. Their analyses confirm the results given in [27].

Jenkins and Gwin [27] also considered data from Finland (1878–1916). They used the data given by Dahlberg [36]. However, Fellman and Eriksson [32] performed a check based on Finnish official registers and confirmed their suspicion that Dahlberg's data contained a misprint for the maternal age group 35 to 40 years. In the analyses, they used the corrected data and present the results in **Figure 7** in [32]. When they applied the linear model to the Finnish data, they obtained the results $\text{TRR} = 0.00003 + 0.742\text{TWR}^2$ and $R^2 = 0.930$. The $SE\left(\widehat{\beta}\right) = 0.091$, $t = -2.8$, and the obtained estimate is significantly below one. The linear relation between $\sqrt{\text{TRR}}$ and TWR is $\sqrt{\text{TRR}} = 0.0026 + 0.768\text{TWR}$ with $R^2 = 0.906$. The $\text{SE}\left(\widehat{\beta}\right) = 0.111$ and $t = -2.1$, and the obtained estimate is significantly below one. All of these results indicate good fit but deficits in triplet maternities [32].

The discrepancies between the results concerning Finnish data given by Fellman and Eriksson and Jenkins and Gwin were mainly caused by two facts; Jenkins and Gwin did not use regression models, but a geometric attempt, and they excluded in their analyses the extreme TRR for the age group 45+ years. In addition, they did not perform any statistical tests. Fellman and Eriksson [32] introduced measures to check both Hellin's law and Jenkins' [35] model in formula (6). They introduced the ratio $\text{HR} = \text{TRR}/\text{TWR}^2$ named Hellin's ratio and assumed that it is a measure of the agreement with respect to Hellin's law. If $\text{HR} > 1$, there is an excess, but if $\text{HR} < 1$, there is a deficit in the TRR. An alternative measure is based on Jenkins' model [32]:

$$\text{J} = \text{TRR} = \frac{1}{n}\sum_{\text{i}} \text{TWR}_{\text{i}}^2 n_{\text{i}} \tag{6}$$

Fellman and Eriksson [29] defined Jenkins' ratio as $JR = \text{TRR}/\text{J}$, where TRR is the total triplet rate. If $\text{JR} > 1$, there are excesses, and if $\text{JR} < 1$, there are deficits in the TRRs. Hellin's ratio can be defined for both age-specific and total rates, but Jenkins' ratio applies only to total rates. In addition, Eq. (6) indicates that JR can be calculated only for data grouped according to maternal age. Based on Schwarz's inequality, a comparison between HR for the total set of maternities and JR yields [32].

$$(\text{TWR})^2 = \left(\frac{1}{n}\sum_{i} (\text{TWR}_{\text{i}})n_{\text{i}}\right)^2 \leq \frac{1}{n}\sum_{i} \left(\text{TWR}_{\text{i}}{}^2 n_i\right) \frac{1}{n}\sum_{i} n_{\text{i}} = \text{J}.$$

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Equality is obtained if and only if

$$\frac{\mathrm{TWR_i}\sqrt{n_\mathrm{i}}}{\sqrt{n_\mathrm{i}}} = \mathrm{TWR_i}$$

for all i. Consequently,

$$\mathrm{HR} = \frac{\mathrm{TRR}}{(\mathrm{TWR})^2} \geq \frac{\mathrm{TRR}}{\mathrm{J}} = \mathrm{JR}.$$

The following step is a simple analysis of the data to show that the transformations may cause excesses in the transformed TRRs and QURs. Fellman and Eriksson [29] simplified their studies by ignoring any random effects. Assume that after the fertilization and any fissions of the fertilized egg, the twinning rate is w_0, the triplet rate is r_0 and the quadruplet rate is q_0, and assume that Hellin's law holds for these rates [32]. Consequently, $r_0 = w_0^2$ and $q_0 = w_0^3$. During pregnancy the rates may decrease, and let the relative reductions be c_w, c_r and c_q for the twinning, triplet and quadruplet rates, respectively. An obvious assumption is that $c_\mathrm{w} \leq c_\mathrm{r} \leq c_\mathrm{q}$. At birth, the observed rates are.

$w = w_0(1 - c_\mathrm{w})$, $r = w_0^2(1 - c_\mathrm{r})$ and $q = w_0^3(1 - c_\mathrm{q})$,

and the variables w, r and q do not satisfy Hellin's law. A fundamental question is whether excesses in the transformed rates of triplets and quadruplets are possible. Compare $w = w_0(1 - c_\mathrm{w})$ and the transformed rates $\sqrt{r} = w_0\sqrt{(1 - c_\mathrm{r})}$ and $\sqrt[3]{q} = w_0\sqrt[3]{(1 - c_\mathrm{q})}$.

An excess for the triplet rate is obtained if $\sqrt{(1 - c_\mathrm{r})} > (1 - c_\mathrm{w})$,

that is, $c_\mathrm{r} < 2c_\mathrm{w} - c_\mathrm{w}^2 \approx 2c_\mathrm{w}$.

An excess for the quadruplet rate is obtained if $\sqrt[3]{(1 - c_\mathrm{q})} > (1 - c_\mathrm{w})$,

that is, $c_\mathrm{q} < 3c_\mathrm{w} - 3c_\mathrm{w}^2 + c_\mathrm{w}^3 \approx 3c_\mathrm{w}$.

These conditions are conceivable, and if the relative reductions in the triplet and quadruplet rates are not too strong, excesses are possible. If one speculates about these results, the extreme excesses observed for transformed quadruplet rates compared with triplet rates, would be explained by the fact that $c_\mathrm{q} < 3c_\mathrm{w}$ is more likely than $c_\mathrm{r} < 2c_\mathrm{w}$ [32]. Consequently, the transformations should be applied with caution and used only for descriptive purposes and not for comparisons between the levels of twinning, triplet and quadruplet rates.

5. Studies including the use of Hellin's law

Fellman and Eriksson [1, 2, 32] presented the temporal trends in TWR, the square root of TRR and the cubic root of QUR obtained from the Veit data [16]. Note that their figure shows stronger fluctuations in TRR than in TWR. However, the confidence bands included indicate that the TWR and the transformed TRR show good agreement for the whole period. The transformed QUR is too high for almost the whole period [32]. In **Figure 1**, we present a new version of the TWR, the transformed TRR and QUR per 10^3 for the Prussian data presented in [16]. In this figure and later, the transformed TRR and QUR per 10^3 are denoted by the initial untransformed names TRR and QUR. Note that the transformed QUR shows a marked excess compared with the TWR and the transformed TRR. This excess can be connected to the comparisons presented above between the rates from conceptions to deliveries. Furthermore, one can observe that all rates show slightly decreasing trends.

In this study, we investigate the temporal trends in TWR, TRR and QUR. The TRRs and QURs are in all figures transformed according to Hellin's law in order to show the association between TWR, TRR and QUR. In the figures, the transformed variables are still denoted TRR and QUR. The trends show variations during different periods and for different countries, but for different countries, one can observe similar patterns. During the eighteenth and nineteenth centuries, the rates are rather similar, but during the first half of the nineteenth century, there is a deficit in the TRR. During the second half of the twentieth century, the TRR shows an excess, and this finding is mainly caused by the influence of the artificial reproduction technologies, particularly the use of fertility-enhancing drugs. Below, we present graphs for different countries, and similar patterns can be noted.

The temporal trends in the TWR and the transformed TRR in Finland 1751–2000 show variations during different periods. During 1750–1900 the rates are rather similar, but during the period 1900–1970, there is a deficit in the TRR. After 1970, the TRR shows an excess, and this finding is mainly caused by the influence of the artificial reproduction technologies, particularly the use of fertility-enhancing drugs (**Figure 2**).

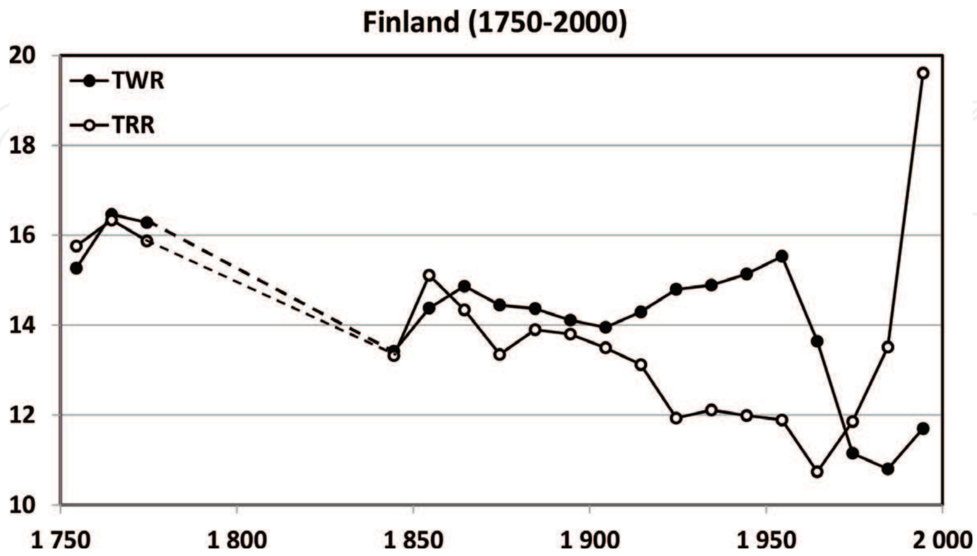

Figure 2. Temporal trends in TWR and transformed TRR in Finland (1751–2000).

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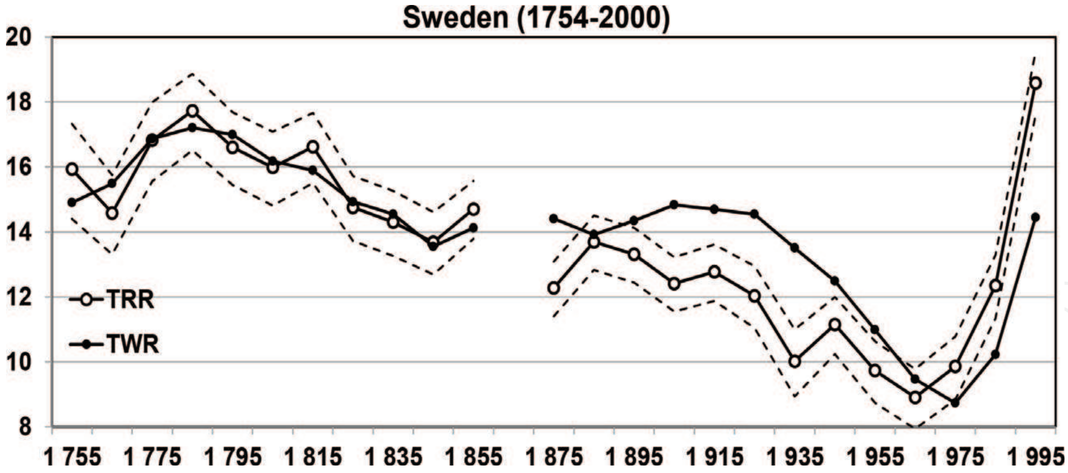

Figure 3. Temporal trends in twinning and triplet rates in Sweden (1751–2000). During 1750–1890 the rates are rather similar, but during the period 1900–1970, there is a deficit in the TRR. After 1970, the TRR shows an excess.

The temporal trends in the twinning and triplet trends in Sweden (1751–2000) are presented in **Figure 3**. During 1750–1890 the rates are rather similar, but during the period 1900–1970, there is a deficit in the TRR. After 1970, the TRR shows an excess.

Following [37] we present in **Figure 4** the temporal trends in the twinning and triplet trends in Portugal (1930–2011). One changing point can be found in 1950. After 1950, TRR shows an excess.

Fellman [3] presented the temporal trends in the twinning and triplet trends in the Netherlands (1950–2003). The findings are given in **Figure 5**. An excess among TRR can be observed after 1970. At the end of the twentieth century, there is a marked deficit in the TRR.

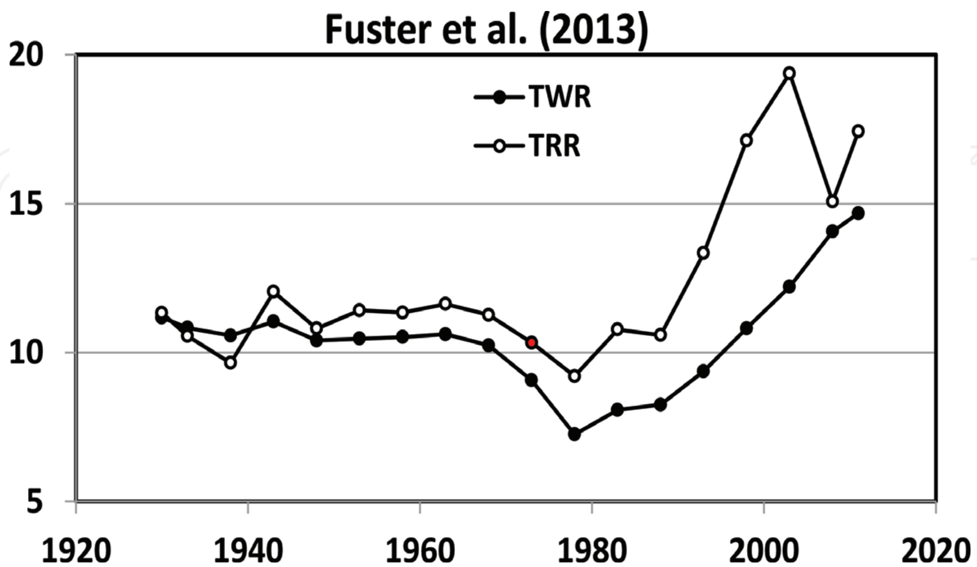

Figure 4. Temporal trends in twinning and triplet trends in Portugal (1930–2011). One changing point can be found at 1950. After 1950, the TRR shows an excess [37].

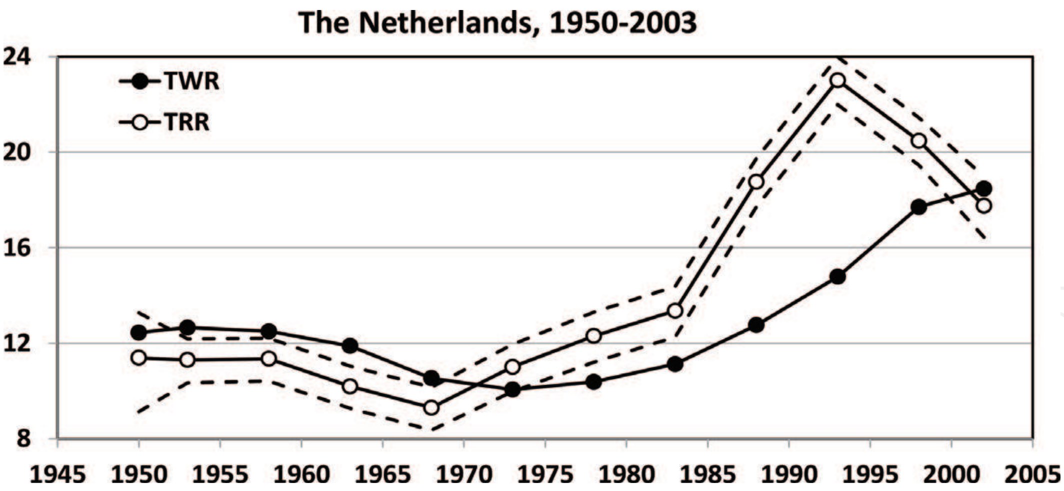

Figure 5. Temporal trends in the twinning and triplet rates in the Netherlands (1950–2003). An excess among TRR can be observed after 1970.

Eriksson and Fellman [33] compared the rates of twin, triplet and quadruplet maternities in England and Wales for the period 1938–2003. In this study, we develop these findings. In **Figure 6**, one observes that before 1970 the graph lines are close to another, but after 1970 the lines rise and diverge. QUR shows the strongest increase and TWR the slightest. Furthermore, **Figure 6** indicates that during the last years, TRR and QUR show a slight decline. Our opinion is that this change is caused by changes in fertilization policies, especially a reduction in the number of fertilized eggs implanted. To clarify the fluctuations, trend lines of sixth degree are included in the figure. Furthermore, **Figure 6** indicates that for data sets after 1970, the TRRs and QURs are markedly too high. It is a remarkable finding that the rates are too high rather

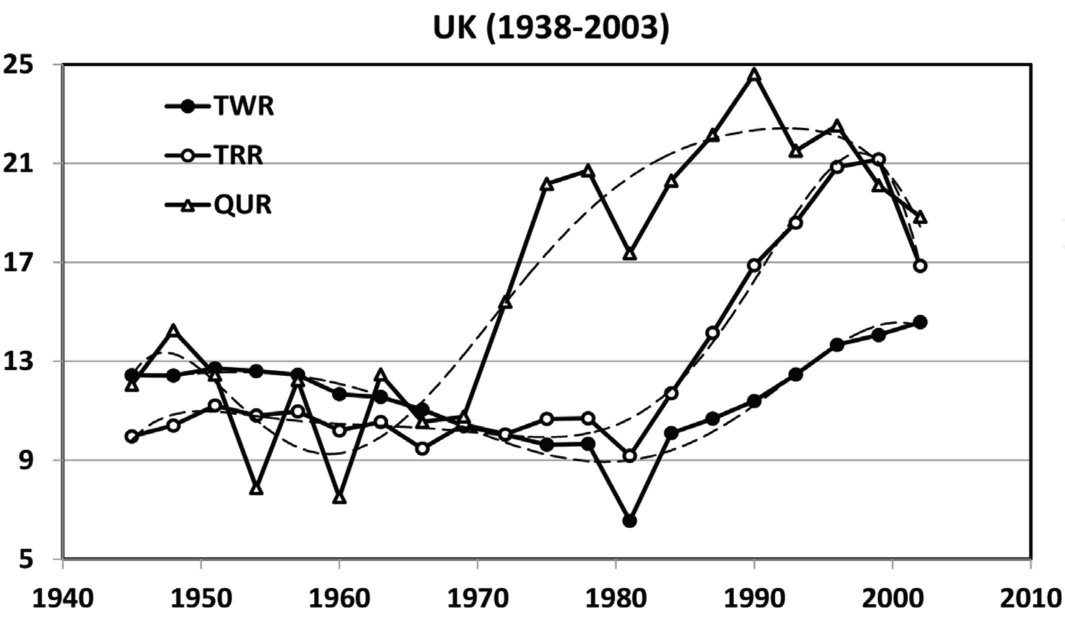

Figure 6. Temporal trends in the twinning, triplet and quadruplet rates in the UK (1938–2003). During 1938–1970 the rates are rather similar, but after 1970 the rates increase. QUR shows the strongest increase and TWR the slightest. In order to clarify the fluctuations, trend lines of sixth degree are included in the figure.

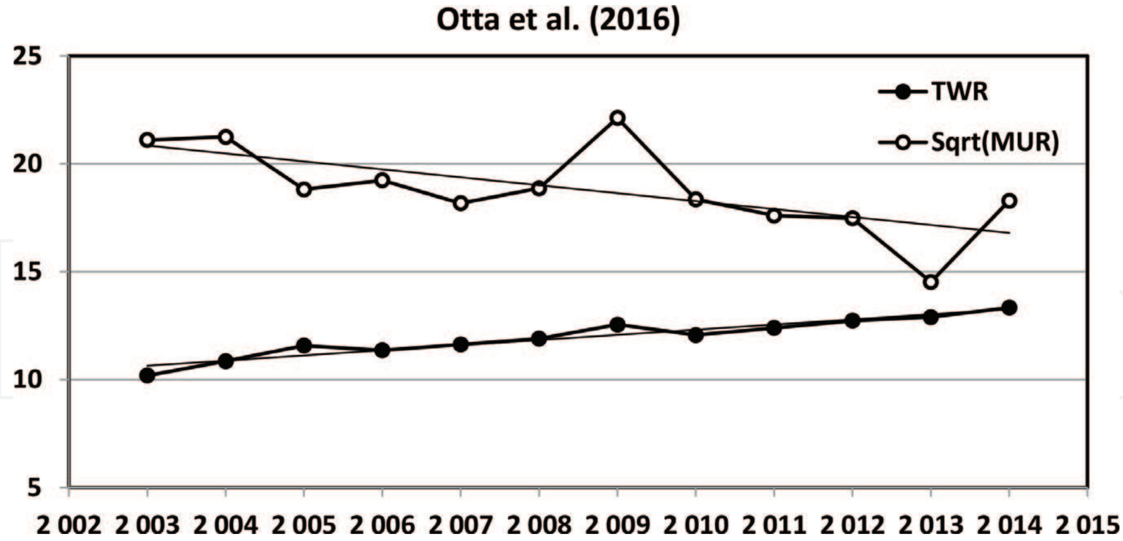

Figure 7. Temporal trends in TWR and the Hellin-transformed rate of multiple maternities (MUR). The figure indicates that the TWR is still increasing, but the transformed MUR is decreasing [38].

than too low, but fertilization policies may result in the extreme sets of multiple maternities. The decreases at the end of the twentieth century are ascribed to changes in the treatment policies discussed above.

Otta et al. [38] analyzed the TWR and the MUR for Brazil data (2003–2014). They discussed the influence of artificial reproduction technologies, particularly the use of fertility-enhancing drugs. They included in their analyses the effect of maternal age. In this study, we look at their data from a different point of view. In **Figure 7**, we present the TWR and the Hellin-transformed rate of multiple maternities (MUR). We assume that the number of multiple maternities is dominated by triplet maternities, and we use the square root transformation. **Figure 7** indicates that the TWR is still increasing, but that the MUR decreases. The excesses coincided with the introduction of subfertility treatments, mainly ovulation inductions. Our opinion is that the difference in the changes between TWR and MUR is caused by changes in fertilization policies, especially a reduction in the number of fertilized eggs implanted. Finally, there is common agreement that discrepancies obtained during the era of fertility treatments are of less interest when Hellin's law is considered because no natural stochastic model is applicable. For the whole period 2003–2014, the TWR = 11.96 per 1000 and MUR = 357.97 per 10^6. Hence, HR = 2.50 indicates a marked excess of multiple maternities.

6. Discussion

A problem that complicates the discussion of Hellin's law is that the law is a mathematical rule concerning theoretical rates, but all checks of the law must be based on empirically obtained rates. In fact, one can only check whether the discrepancies are too large and cannot be explained by random errors. Although the discrepancies are small, Hellin's law cannot be accepted as a theoretical one. In this way, no exact proof to support the law can be obtained [32].

Jenkins ([35], Figure 8) and later Fellman and Eriksson [1] presented the association between the TWRs and the TRRs in the Prussian data [16]. They observed marked fluctuations for the different annual data but a good agreement between the TRR and the TWR for the total data set [32].

Our impression is that the finding already noted by Strassmann [20] was the birth of Hellin's law. Furthermore, Jenkins [35] stressed that Hellin's law is a first approximation. It is generally agreed that the main argument for Hellin's law is that the probabilities of additional ovulations and the fissions of fertilized eggs can be explained by stochastic models. Consequently, in large data sets, the averages could be stable and formulated by a mathematical relation (Hellin's law). A common argument for the discrepancies is that after the conceptions, there is a long process influenced by disturbing factors (intrauterine deaths, spontaneous abortions, etc., of one or more fetuses). Jenkins [35] and Komai and Fukuoka [39], for instance, assumed that differential mortality in utero of twins and triplets could be one such factor. Consequently, the final result often shows only a weak resemblance to the outcome of a simple stochastic process associated with the initial conceptions. Excesses of higher multiple maternities in old birth registers must be considered paradoxical. One explanation can be the results of the comparisons between the changes in the rates of singletons, twins and triplets during the time from conception to confinement discussed above. Another probable explanation is that systematic errors in the registers may cause biases in the data. This explanation is less plausible if the data are collected in different countries, as is the case in data in ([32, 35], **Table 1** and **Figure 1**).

In his study of the rates of multiple maternities for total, "white" and "colored" in US populations (1922–1936), Strandskov [40] evaluated how well his data satisfy Hellin's law. Applying χ^2 tests, he found that in none of the populations tested did the observed plural birth frequencies agree closely with Hellin's law [3].

Based on hospital data, Sarkar [41] studied the TWR in India and on Ceylon (Sri Lanka). His paper is interesting because he defined the TWR as $1 : n$ and the triplet rate as $1 : m^2$, that is, he indirectly used a modified Hellin's law without any reference to Hellin. One finds a deficit of triplet maternities ($m > n$). In addition, one observes that on Ceylon the TWR was low ($1 : 161.1$), yielding a TWR of 6.21 per 1000. On Ceylon, the TRR followed Hellin's law more exactly because it was $1 : 154.4^2$.

Das [42] formulated Hellin's law such that "the frequency of twin confinements bears to that of total confinements a ratio which is equal to the ratio borne by the frequency of the triplet confinements to that of the twin confinements". This modified definition is in congruence with Strassmann's version of the law. He reviewed earlier studies concerning Hellin's law and stressed the discrepancies presented in them [3]. Das concluded that Hellin's law has no sound basis and that exceptions to the rule have been the rule. In a later paper, Das [43] also considered the relation $\mathrm{TRR} = (\mathrm{TWR})^2$. He constructed an advanced model based on the zygosity of both twins and triplets. His mathematical analyses of these models did not support Hellin's law [1].

Fellman and Eriksson [2] compared in **Figure 3** the TWR and the transformed TRR and QUR for Sweden (1751–2000). For the period 1871–1960, there is a deficiency in the TRR. Fellman and Eriksson [32] discuss this deficiency in more detail. There is almost constantly an excess in the QUR for the whole period. After 1970, both the TRR and QUR show excesses, but this is mainly caused by the influence of the artificial reproduction technologies, particularly the use of fertility-enhancing drugs. For references, see [14, 32].

Above the HR is defined as HR = TRR/TWR^2. Agreements between TWR and transformed TRR can show remarkable variations. Eriksson [12] studied the TWR and the TRR in the southwestern part of Finland. On the Åland islands, the TWR was continuously high. For the period 1653–1949, the TWR was 19.21 per 1000, and the TRR was 375 per 10^6. According to Hellin’s law, the expected TRR was 369 per 10^6. For the Åland data, HR = 1.02, showing a good agreement with Hellin’s law. In the Åboland (Turunmaa in Finnish) archipelago, close to the Åland islands, the TWR was also high. For the period 1655–1949, the TWR was 20.90 per 1000. For the same period, the TRR was 252 per 10^6. According to Hellin’s law, the expected TRR was 437 per 10^6, yielding HR = 0.58, and consequently, the Åboland archipelago data showed a marked deficit in TRR with respect to Hellin’s law [32].

Lam and Ho [44] noted an increase in the number of multiple maternities in Hong Kong in 1981–1995. They also stressed the marked discrepancy between the observed data and Hellin’s law. Zhang et al. [45] have observed similar increases in the rates of multiple maternities among older mothers in the USA in 1995–1997, and they also attributed this finding to the increased use of assisted reproductive technology. Simmons et al. [46] noted a dramatic decrease in the proportion of triplet and higher-order births since 1998 [32].

7. Conclusion

It is generally agreed that the main argument for Hellin’s law is that the probabilities of additional ovulations and the fissions of fertilized eggs can be explained by stochastic models. Consequently, in large data sets, the averages could be stable and formulated by a mathematical relation (Hellin’s law). A common argument for the discrepancies is that after the conceptions, there is a long process influenced by disturbing factors (intrauterine deaths, spontaneous abortions, etc., of one or more fetuses) [32]. The discussion of Hellin’s law is complicated by the fact that the law is a mathematical rule concerning theoretical rates, but all checks have to be based on empirically obtained rates. In fact, one can only check if the discrepancies are so large that they cannot be explained by random errors. If the discrepancies are small, an exact Hellin’s law cannot be accepted. In this way, no exact proof to support the law can be obtained [32].

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Conflict of interest

No conflict of interest.

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